# Complementary Local and Extended Views of Bonding in the $\mathrm{ThCr}_{2} \mathrm{Si}_{2}$ and $\mathrm{CaAl}_{2} \mathrm{Si}_{2}$ Structures* 

CHONG ZHENG<br>Department of Chemistry, Stanford University, Stanford, California 94305

and ROALD HOFFMANN
Department of Chemistry and Materials Science Center, Cornell University, Ithaca, New York 14853

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Local and extended constructions of the bonding in transition metal $A B_{2} X_{2}$ compounds of $\mathrm{ThCr}_{2} \mathrm{Si}_{2}$ and $\mathrm{CaAl}_{2} \mathrm{Si}_{2}$ types are contrasted. From the local perspective, in the "inverted" $X B_{4}$ tetrahedral coordination observed in the $\mathrm{CaAl}_{2} \mathrm{Si}_{2}$ structure, the greater topological flexibility of the transition metal ( $B$ ) $d$ orbitals allow for better adjustment to the strain. Thus the difference between the strength of the 1 -fold and the 3 -fold $B-X$ bonds is small compared to the main group analogs. From the extended structure point of view, the metal sublattice in the $\mathrm{CaAl}_{2} \mathrm{Si}_{2}$ structure is less "dispersive" than in the $\mathrm{ThCr}_{2} \mathrm{Si}_{2}$ alternative. Hence, the metals interact better with the ligand sublattice at low ( $\sim d^{0}$ ) and high $\left(\sim d^{10}\right)$ electron counts. This may be one of the reasons the $\mathrm{CaAl}_{2} \mathrm{Si}_{2}$ structure occurs only for $d^{0}, d^{5}$, and $d^{10}$ configurations at the transition metal. © 1988 Academic Press, Inc.

## Introduction

The $\mathrm{ThCr}_{2} \mathrm{Si}_{2}$ structure, 1, accounts for more than 400 compounds of the $A B_{2} X_{2}$


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* Dedicated to John B. Goodenough with admiration for his insight into the electronic structure of extended materials.
composition (1). In these, $A$ is typically a rare earth, an alkaline earth, or an alkali element, $B$ is a transition metal or main group element, and $X$ is an element of Group 15,14 , and occasionally 13.

Another prototype, $\mathrm{CaAl}_{2} \mathrm{Si}_{2}, \mathbf{2}$, is probably the runner-up in the structural popu-

larity competition in the $A B_{2} X_{2}$ series. Dozens of solids crystallize in the $\mathrm{CaAl}_{2} \mathrm{Si}_{2}$ mold (2). The apparent requirement that a legitimate $A B_{2} X_{2}$ candidate for the $\mathrm{CaAl}_{2} \mathrm{Si}_{2}$ $A B_{2} X_{2}$ structure have a $d^{0}, d^{5}$, or $d^{10}$ configuration (2c) at atom $B$ seems to limit the occurrence of this structural type. We should make the electron-counting scheme we use more precise. For example, if we take $\mathrm{CaAl}_{2} \mathrm{Si}_{2}(2 a)$ and assign oxidation states as $\mathrm{Ca}^{2+}\left(\mathrm{Al}^{3+}\right)_{2}\left(\mathrm{Si}^{4-}\right)_{2}$, we get a formal $d^{0} \mathrm{Al}^{3+}$, while $\mathrm{Ca}^{2+}\left(\mathrm{Mn}^{2+}\right)_{2}\left(\mathrm{P}^{3-}\right)_{2}$ and $\mathrm{Ca}^{2+}\left(\mathrm{Zn}^{2+}\right)_{2}\left(\mathrm{P}^{3-}\right)_{2}$ exemplify the $d^{5}$ and $d^{10}$ cases. It should be mentioned that in general the fact that there is no short $X \cdots X$ contact in the $\mathrm{CaAl}_{2} \mathrm{Si}_{2}$ structure allows us to count Si and P as $\mathrm{Si}^{4-}$ and $\mathrm{P}^{3-}$.

Thus, we are presented with this question: Why does a $\mathrm{CaAl}_{2} \mathrm{Si}_{2}$-type compound have a $d^{0}, d^{5}$, or $d^{10}$ configuration at the atom $B$ ? And to approach an answer to this question we need to understand the structural details of both $\mathrm{ThCr}_{2} \mathrm{Si}_{2}$ and $\mathrm{CaAl}_{2} \mathrm{Si}_{2}$ structures. This we proceed to do with the assistance of calculations of the extended Hückel type. The computations do not take spin or magnetic interaction into account. But with the transparency of the method and our experience in this $A B_{2} X_{2}$ series (3), we hope to gain some insight into the nonmagnetic part of the forces that hold the solids together. The details of the calculations are given in the Appendix.

## The $\mathbf{T h C r}_{2} \mathbf{S i}_{2}$ and $\mathbf{C a A l}_{2} \mathbf{S i}_{2}$ Structures

The $\mathrm{ThCr}_{2} \mathrm{Si}_{2}$ and $\mathrm{CaAl}_{2} \mathrm{Si}_{2}$ structures are shown in 1 and 2, respectively. Both stoichiometric $A B_{2} X_{2}$ compounds consist of interspersed $B_{2} X_{2}^{n-}$ layers and $A^{n+}$ (mostly alkaline earth or rare earth) cation layers. The interactions between the cations and the anionic layers are predominantly ionic and should contribute very little to the electronic energy in which we are interested. That is why we will ignore the cations in our analysis and consider them
merely as electron donors, adding extra electrons into the host $B_{2} X_{2}$ layer. However, the ionic Madelung contribution is surely important in determining the stability of alternative structures. It could be superimposed on the electronic energy, and the reader is referred to our earlier qualitative analysis of this effect ( $3 b$ ).

The split $B_{2} X_{2}$ layers of the $\mathrm{ThCr}_{2} \mathrm{Si}_{2}$ and the $\mathrm{CaAl}_{2} \mathrm{Si}_{2}$ structures are shown in $\mathbf{3}$ and 4 , respectively. The $B \cdots X$ contact is


around $2.5 \AA$ and the $X \cdots X$ one is very long, usually greater than $4 \AA$. Locally $B$ resides at the center of an $X_{4}$ tetrahedron 5


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in both structures, and these tetrahedra pack differently to form the two types of crystals. In 6, each tetrahedron shares four of its six edges with others; in 7 it shares three.


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Given that the local environment of a $B$ atom is the same tetrahedral one in both structures, it must be the extended, nonlocal interactions that favor one structure over the other. But before we explore the bonding from an extended point of view, let us look at the local bonding interactions in more detail.

## A Local View of Bonding

The $d^{0}$ and $d^{10}$ configurations of $B$ atoms in the $\mathrm{CaAl}_{2} \mathrm{Si}_{2}$ structure are relatively easy to rationalize in terms of the Zintl concept. An electron count of 0 or 10 for $B$ in $A B_{2} X_{2}$ is coupled in our counting scheme with a full octet for $X$. We thus come to 8 or 18 electrons per $B X$ (e.g., $\left[\mathrm{Al}^{3+} \mathrm{Si}^{4-}\right]^{-}$in $\mathrm{CaAl}_{2} \mathrm{Si}_{2}$ ). This electron count is identical to that of C or ZnS , and so one might expect a similar three- or four-connected structure.

In fact, one does get a four-connected net in $\mathrm{CaAl}_{2} \mathrm{Si}$, but it is not that of the diamond or wurtzite structure. The Al is at the center of a normal $\mathrm{Si}_{4}$ tetrahedron, $\mathbf{8}$; Si , however, is in an environment of an "inverted" $\mathrm{Al}_{4}$ tetrahedron, 9.


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What about the the alternative $\mathrm{ThCr}_{2} \mathrm{Si}_{2}$ $A B_{2} X_{2}$ structure? The Cr , or $B$ atom, is tetrahedrally surrounded by four $X$ (and has also four close $B$ neighbors). The $X$ is also four-coordinate, but with the unusual square-pyramidal geometry. A special and fascinating feature of $\mathrm{ThCr}_{2} \mathrm{Si}_{2}$ structures is an interlayer $X-X$ contact that often becomes a bond. In most main group examples of this structure (e.g., $\mathrm{BaAl}_{4},{R E A I_{2}}^{2}$ $\mathrm{Ga}_{2}$ ) this interlayer bond is present and strong. It makes $X$ five-coordinate. Clearly, simple Zintl ideas do not apply. One needs to invoke and analyze the multi-center bonding that is present. We have done this in a previous contribution ( $3 a$ ) and have shown the similarity of the electron-deficient multi-center bonding in $\mathrm{BaAl}_{4}$ to that in molecular $\mathrm{B}_{5} \mathrm{H}_{9}$ or $\mathrm{B}_{10} \mathrm{H}_{16}$. The appropriate electron count is 14 per $B_{2} X_{2}$ or 7 per $B X$. Putting 8 electrons per $B X$ into this structure would fill some antibonding levels, so it is not surprising that the compounds under discussion in this paper prefer the four-connected Zintl structure of $\mathrm{CaAl}_{2} \mathrm{Zn}_{2}$.

Let's examine in a little more detail the bonding in this structure. Schematically the bonding picture is that of $\mathbf{1 0}$. Our previous

analysis (3c) shows that the HOMO of $\mathbf{1 0}$ can be described as a mixture of two orbitals derived from the LUMO and HOMO of

a perfect $\mathrm{SiAl}_{4}$ tetrahedron. The mixing, 11, weakens the vertical "handle"' Si - Al bond, and in all the known $d^{0}$ and $d^{10} \mathrm{CaAl}_{2} \mathrm{Si}_{2^{-}}$ type compounds that bond is longer than the "rib" $\mathrm{Si}-\mathrm{Al}$ bond.

The $d^{5}$ compounds, however, seem to be different. Since they are going to be the main subject of our discussion, we list some of these compounds in Table I. In most of the $d^{5} \mathrm{Mn}$ compounds in Table I, the difference in length between the "rib" and the "handle" bonds is small, and in some the "handle" bond is even shorter.

Let us study this geometrical distinction (or lack of it) in the $d^{5}$ structures with a model molecular $\mathrm{PMn}_{4}$ cluster. We will begin with an ideal tetrahedron around P , and then deform it, 12.


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A schematic orbital diagram for the ideal $\mathrm{PMn}_{4}$ tetrahedron is shown in 13. The lowest four orbitals are mainly phosphorus $s$ and $p$ orbitals combined with symmetryadapted Mn local radial $\sigma$ orbitals ( $d_{z^{2}}+s+$ $p_{z}$ ). Above these four orbitals are the 20 Mn $d$-block orbitals. Their nature matches expectations. At the bottom of the $d$ block, the orbitals have mainly local $\mathrm{Mn} \delta$ and $\pi$ character and thus are $\mathrm{Mn}-\mathrm{P}$ nonbonding. At the top of the block, one finds part of the antibonding counterparts to the lowest four orbitals of the scheme (the ones which are $\mathrm{Mn}-\mathrm{P}$ bonding, mostly on P ). Actually, the
major part of $\mathrm{Mn}-\mathrm{P}$ antibonding is in still higher orbitals, originating from $\mathrm{Mn} 4 s, 4 p$.

Now imagine the flipping process, 12. The $s, p_{x}$, and $p_{y}$ orbitals of phosphorus retain their overlaps with metal orbitals and thus remain approximately the same in energy. But $p_{z}, \mathbf{1 4}$, increases in energy because of antibonding between the $p$ and rib Mn orbitals. This orbital actually rises to the top of the $d$ block. Replacing it is another orbital, originally of local $\mathrm{Mn} \pi$ character, 15, coming down from the $d$ block. There are many Mn $d$ orbitals at the same energy (as compared to only one $s$ orbital at each Al in the main group $\mathrm{CaAl}_{2} \mathrm{Si}_{2}$ case) and thus more topological flexibility in $\mathrm{PMn}_{4}$ compared to $\mathrm{SiAl}_{4}$. The bonding interactions in 15 can adjust better to the deformation; the rib bonds need not weaken so much.

In our extended Hückel calculations, we follow the bond strength by carrying out calculations on a model with symmetrynonequivalent (here rib and handle) bonds set equal in length, and then we look at the unequal overlap populations that result as indicators of bonding changes that will take place. The Mn containing $\mathrm{CaAl}_{2} \mathrm{Si}_{2}$ phases present a special problem in that we do not know, experimentally, the spin state of Mn in them. $\mathrm{Mn}(\mathrm{II})$ is, of course, an ideal candidate for a high-spin magnetic solid. We think one must consider, therefore, two possible situations, drawn schematically in 16a and 16b. In 16a, the low-spin case, the bottom of the $d$ band is occupied by two electrons per orbital. In 16b, the high-spin alternative, there is one electron per band. Our calculations unfortunately do not include spin explicitly, and so cannot deter-





13


14


15

a

b
mine the energetics of these alternatives. In the remainder of the paper we will usually give the results for both alternative cases.

Returning to the $\mathrm{PMn}_{4}$ model, if we begin
with $d^{0}$, the $d$ band empty, the overlap populations still indicate a stronger handle bond, but compared to the main group case ( $3 c$ ), the bond strength differential is substantially smaller. Filling of the $d$ block up
to $d^{5}$ (low spin) does not change the differential much, since those states filled are mostly $\mathrm{Mn}-\mathrm{P}$ nonbonding. If we choose instead the high-spin filling, one electron per band in the entire $d$ band, then both axial and rib bonds are weakened. The orbitals at the top of the $\mathrm{Mn} d$ band are $\mathrm{Mn}-\mathrm{P}$ antibonding. The actual overlap populations for low-spin and high-spin $d^{5}$ fillings, compared to those of the main group $\mathrm{SiAl}_{4}^{8+}$ cluster, are shown in 17. The


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difference between the rib and handle bond overlap populations is 0.156 (low spin) and 0.132 (high spin), compared to 0.232 in the main group case.

Let us go to the two-dimensional $\mathrm{Mn}_{2} \mathrm{P}_{2}^{2-}$ layer. In the infinite system, we have instead of a single energy level a bunch of states, or a density of states in the language of the trade. The overlap population equivalent is the COOP curve, shown in Fig. 1. The COOP curves are really overlap pop-ulation-weighted densities of states and gauge the bonding capabilities of all the energy levels in a given energy interval (9). They are the solid state analog of an overlap population, with positive values indicating bonding, negative ones antibonding. Thus, the states around -20 and -15 eV , mostly phosphorus $s$ and $p$, are $\mathrm{Mn}-\mathrm{P}$ bonding. The states from -12 to -8 eV are the $\mathrm{Mn} d$ band. The lower half of the band is mainly $\mathrm{Mn}-\mathrm{P}$ nonbonding and upper half antibonding. The handle $\mathrm{Mn}-\mathrm{P}(1 \times)$ bond is more antibonding in the upper part of the


Fig. 1. Crystal orbital overlap population (COOP) curves for the two types of $\mathrm{Mn}-\mathrm{P}$ bonds in $\mathrm{Mn}_{2} \mathrm{P}_{2}^{2-}$ ( $\mathrm{CaAl}_{2} \mathrm{Si}_{2}$ type). The solid curve is for the 1 -fold $\mathrm{Mn}-\mathrm{P}$ bond and the dotted one for the 3 -fold $\mathrm{Mn}-\mathrm{P}$ bond.
$d$ band. All of these features are expected from our discussion concerning 13,14 . The calculated overlap population for this infinite two-dimensional $\mathrm{Mn}_{2} \mathrm{P}_{2}^{2-}$ layer is shown in 18. As in the case of the $\mathrm{PMn}_{4}$


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cluster, the difference of 0.110 low spin and 0.146 high spin between the rib and handle bonds is substantially smaller than in the corresponding two-dimensional main group compound. Note that the high-spin occupation weakens the rib bond less than it does the handle bond, as expected from schematic 13.

Thus, we may conclude that for a $d^{5}$ compound of the $\mathrm{CaAl}_{2} \mathrm{Si}_{2}$ type, the bond length difference between the rib and

TABLE I
Some $d^{5} \mathrm{CaAl}_{2} \mathrm{Si}_{2}$-Type Compounds

| $A B_{2} X_{2}$ <br> compound | Bond length ( $\AA$ ) |  | Ref. |
| :---: | :---: | :---: | :---: |
|  | $\begin{aligned} & \text { Rib or } \\ & 3 \times B-X \end{aligned}$ | Handle or $1 \times B-X$ |  |
| $\mathrm{CaMn}_{2} \mathrm{As}_{2}$ | 2.585 | 2.587 | 5 |
| $\mathrm{SrMn}_{2} \mathrm{As}_{2}$ | 2.610 | 2.583 | 5 |
| $\mathrm{CaMn}_{2} \mathrm{Sb}_{2}$ | 2.781 | 2.782 | 6 |
| $\mathrm{SrMn}_{2} \mathrm{Sb}_{2}$ | 2.757 | 2.785 | 6 |
| $\mathrm{CaMn}_{2} \mathrm{Bi}_{2}$ | 2.854 | 2.852 | 6 |
| $\mathrm{CaMn}_{2} \mathrm{P}_{2}$ | 2.491 | 2.488 | 7 |
| $\mathrm{SrMn}_{2} \mathrm{P}_{2}$ | 2.525 | 2.476 | 7 |
| $\mathrm{SrMn}_{2} \mathrm{As}_{2}$ | 2.619 | 2.585 | 7 |
| YbMnCuP 2 | 2.411 | 2.472 | 8 |

handle bonds is smaller, compared to a $d^{0}$ or $d^{10}$ structure, as we have seen in Table I. This is consistent with the experimental observations.

## The $\mathbf{T h C r}_{2} \mathbf{S i}_{2}$ Structure

A bonding scheme for the $\mathrm{ThCr}_{2} \mathrm{Si}_{2}$ structure $\mathbf{3}$ can be constructed similarly by first
examining the local bonding and then turning on extended interactions. For the main group compounds ( $\mathrm{BaAl}_{4}, R E \mathrm{Al}_{2} \mathrm{Ga}_{2}$, etc.) we have shown in a previous contribution (3a) that the bonding scheme is very similar to that of $\mathrm{B}_{5} \mathrm{H}_{9}$ or $\mathrm{B}_{10} \mathrm{H}_{16}$. Starting with a basis set of the $s p^{3}$ hybrids of the basal atom and the $p_{x}, p_{y}$, and $s p$ hybrids of the apical atom, 19, one forms cyclo-


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butadienoid symmetry-adapted linear combinations of the $s p^{3}$ orbitals to interact with the apical orbitals. This occurs at each hollow of the square lattice, $\mathbf{2 0}$. The local bonding scheme, in which the apical $h y_{\text {in }}$, $p_{x}, p_{y}$ interact with the basal $a_{1}$ and $e$ MO's, respectively, produces three apical-basal bonding orbitals and one nonbonding one.


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In three dimensions, two such hollow sites and the two $h y_{\text {out }}$ orbitals from each form a bonding and an antibonding combination. This scheme accounts for the magic 14 electron per unit cell count in $\mathrm{BaAl}_{4}$-type compounds. Even after the delocalized interaction is turned on and each orbital develops into a band, the local character responsible for this electron count remains. The construction of the three-dimensional bands is shown schematically in 21.


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For the transition metal case we can derive the approximate bands in a similar way. The local tetrahedral interaction splits the metal $d$ orbitals into $e$ and $t$ sets, and when the extended interaction is turned on, both $e$ and $t$ sets develop into bands. This is shown schematically in 22 . Thus we expect that the $d$ band region is metal-ligand antibonding, more so at its top. The bottom of the $d$ band should be metal-metal bonding and the top metal-metal antibonding. Our expectation is confirmed by the calculated COOP curves in Fig. 2, where the Mn-P antibonding and the $\mathrm{Mn}-\mathrm{Mn}$ bonding/ antibonding features between -12 and -9 eV are clearly seen.

$\begin{array}{ll}\text { Separate atoms } & \begin{array}{l}\mathrm{Mn}-\mathrm{P} \text { bonding } \\ \text { turned on }\end{array}\end{array}$
Extended interactions, especially Mn... Mn bonding, turned on

## Delocalized Construction

Let us approach the construction of the energy bands from another point of view. Imagine that the ligands (phosphorus, in our case) are stripped off from the layer lattice 3 and 4. The bare metal layers arc then a square lattice, 23 , and an $A B$ closedpacked lattice, 24. In the square lattice,


Fig. 2. COOP curves for the $\mathrm{Mn}-\mathrm{Mn}$ (solid line) and $\mathrm{Mn}-\mathrm{P}$ (dotted line) bonds in $\mathrm{Mn}_{2} \mathrm{P}_{2}^{2-}\left(\mathrm{ThCr}_{2} \mathrm{Si}_{2}\right.$ type).


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each metal has four nearest neighbors and in the $A B$ lattice, three. Thus for the same metal-metal distances (as assumed in our calculation), the $d$ band dispersion is larger in 23 than in 24. A band of greater dispersion has associated with it both strong bonding and antibonding at the botiom and top, respectively. And a little-dispersed band implies weaker nearest-neighbor interactions both bonding and antibonding. These situations are contrasted in 25.

The energetic consequences of dispersion are a function of band filling. If the bands are completely filled, it might seem at first sight that there should be no energy difference between the two lattices. But this is not so. Just as in molecules (the corresponding phenomenon there is fourelectron two-orbital repulsion), the antibonding at the top of the band is stronger than the bonding at the bottom. This is a consequence of including overlap in the calculations.

Following this line of reasoning, at low band filling, the lattice with larger band dispersion should be preferred, a conse-
quence of the strong bonding feature at the band bottom. But for high filling, the less dispersed band should be favored, avoiding the strong antibonding feature at the band top. Thus for our bare metal lattices, 23 and 24, one would expect that the square lattice should be more stable at low $d$ electron counts and the $A B$ lattice at high $d$ band filling.

The actual energetic difference computed between the two bare metal lattices from an extended Hückel calculation is shown in Fig. 3. At low $d$ electron counts, indeed, the square lattice is favored, but at high band filling, it is again the square net which has lower energy. A detective decomposition of the $M-M$ COOP curve (not shown here) tells us that this preference at the high electron filling is due to the metal $s$, $p$ states, which penetrate into the top of the $d$ band. Those states are of strong $M-M$ bonding character for a more dispersive lattice. A picture, 26, accounts for the shape of the curve in Fig. 3. In other words, higher dispersion is favored at high $d$ electron counts, since the states populated


Fig. 3. Energy difference between the two metal sublattices of $\mathrm{Mn}_{2} \mathrm{P}_{2}^{2-}\left(\mathrm{CaAl}_{2} \mathrm{Si}_{2}\right.$ and $\mathrm{ThCr}_{2} \mathrm{Si}_{2}$ types). $\Delta E$ is defined as $E\left(\mathrm{ThCr}_{2} \mathrm{Si}_{2}\right)-E\left(\mathrm{CaAl}_{2} \mathrm{Si}_{2}\right)$; thus a positive $\Delta E$ means that the sublattice in the $\mathrm{CaAl}_{2} \mathrm{Si}_{2}$ type is more stable than that in the $\mathrm{ThCr}_{2} \mathrm{Si}_{2}$ type.


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arose from the hybridization of the bottom of the $s, p$ band with metal $d$.

Note that the shape of Fig. 3 corresponds to a typical fourth moment curve (10). A referee pointed out that it is smaller to the published curve of Burdett and Lee for $\alpha$-bismuth and polonium (10) and could be interpreted in a similar way.

Now let us bring up to the metal sublattice the phosphorus sublattice, 27, and

a

b


Fig. 4. COOP curves of the $\mathrm{Mn}-\mathrm{Mn}$ bond in the $\mathrm{Th} \mathrm{Cr}_{2} \mathrm{Si}_{2}$ structure as that structure is assembled by bringing up the phosphorus sublattice to the transition metal sublattice, (a) $\mathrm{Mn}-\mathrm{P}=5.0 \AA$; (b) $\mathrm{Mn}-\mathrm{P}=$ $3.5 \AA$; (c) $\mathrm{Mn}-\mathrm{P}=3.0 \AA$; (d) $\mathrm{Mn}-\mathrm{P}=2.45 \AA$ (ideal tetrahedral coordination).
follow the evolution of the bonding. Figure 4 shows the change of the COOP curve for the $\mathrm{Mn}-\mathrm{Mn}$ bond in the $\mathrm{ThCr}_{2} \mathrm{Si}_{2}$ structure, 27a, when we bring in the $P$ sublattice from far $(\mathrm{Mn}-\mathrm{P}=5 \AA)$ to the final ideal tetrahedron coordination ( $\mathrm{Mn}-\mathrm{P}=2.5 \AA$ ). At a $\mathrm{Mn}-\mathrm{P}$ separation of $5 \AA$, there is no interac-
tion between the metal and ligand sublattices, and the COOP curve (Fig. 4a) is basically that of the bare square Mn lattice (23). At the bottom of the $d$ band $(-13 \mathrm{eV})$, it is $\mathrm{Mn}-\mathrm{Mn}$ bonding. At the top ( -10 eV ) it is weakly antibonding, due to the mixing with the bottom of the $s, p$ band $(-8 \mathrm{eV})$.

At a closer $\mathrm{Mn}-\mathrm{P}$ separation of $3.5 \AA$, the COOP (Fig. 4b) remains more or less the same, except that a small bump at -14 eV develops. This is a resonance with the phosphorus $p$ band.

A $\mathrm{Mn}-\mathrm{P}$ distance of $3 \AA$ is within the interacting range, and we expect the COOP curve to change a lot. In Fig. 4c, we see that (1) a large portion of the states of $\mathrm{Mn}-\mathrm{Mn}$ bonding character is pushed up, and (2) two resonant bumps at phosphorus $s(\sim-19 \mathrm{eV})$ and $p(\sim-14.5 \mathrm{eV})$ become more obvious. The states in these energy ranges are mainly $\mathrm{P} 3 s$ and $3 p$ and are primarily engaged in $\mathrm{Mn}-\mathrm{P}$ bonding. But note that $\mathrm{Mn}-\mathrm{Mn}$ bonding is picked up in these same bands.

We interpret the above observations as follows. Those metal states which are $\mathrm{Mn}-\mathrm{Mn}$ bonding have, of course, fewer nodes in the metal sublattice than the $\mathbf{M n}-\mathrm{Mn}$ antibonding states. The approaching $P$ ligands are coming down into the triangular or square hollows and there would sample effectively the presence or absence of nodes. Metal-P interaction is thus expected to be better for the metal states at the bottom of the metal band. These metal states divide after interaction into two parts: $\mathrm{Mn}-\mathrm{P}$ bonding (resonating with phosphorus $s$ and $p$ levels) and $\mathrm{Mn}-\mathrm{P}$ antibonding (pushed up). States which are strongly $\mathbf{M n}-\mathrm{Mn}$ antibonding interact less effectively with phosphorus and thus remain where they were at the energy scale. This is made clear by following the constant $\mathrm{Mn}-\mathrm{Mn} d$ block antibonding feature in Fig. $4 a \rightarrow 4 d$.

Figure 5 shows the metal-metal overlap population differences between the two crystal structures (overlap population of $\mathrm{CaAl}_{2} \mathrm{Si}_{2}$ minus that of $\mathrm{ThCr}_{2} \mathrm{Si}_{2}$ ). The solid curve is the $\mathrm{Mn}-\mathrm{Mn}$ overlap population when the phosphorus sublattice is far away ( $5 \AA$ ). Since there is no $\mathrm{Mn}-\mathrm{P}$ interaction, the curve should parallel, roughly, the energy curve for the bare lattices in Fig. 3.


Fig. 5. Mn-Mn overlap population (OP) difference between the $\mathrm{CaAl}_{2} \mathrm{Si}_{2^{-}}$and $\mathrm{ThCr}_{2} \mathrm{Si}_{2}$-type $\mathrm{Mn}_{2} \mathrm{P}_{2}^{2-}$ as the structures are assembled from metal and ligand sublattices. The difference is defined as $\mathrm{OP}\left(\mathrm{CaAl}_{2} \mathrm{Si}_{2}\right)$
$\mathrm{OP}\left(\mathrm{ThCr}_{2} \mathrm{Si}_{2}\right)$. Thus a positive value implies a stronger $\mathrm{Mn}-\mathrm{Mn}$ bond in the $\mathrm{CaAl}_{2} \mathrm{Si}_{2}$-type lattice.

In the ideal tetrahedral coordination (each phosphorus has four nearest metal neighbors in both structures), the overlap population difference (dashed curve) is "inverted." The $M-M$ bond becomes weaker in one lattice at the $d$ electron count where it was stronger, and vice versa, when the $M-\mathrm{P}$ interaction is turned on. This provides additional evidence for our conclusion that metal states which are more bonding interact more strongly with ligand states and thus are pushed up (emptied). Since locally metal-ligand coordination is the same in both structures, we expect that the metal-metal interaction which determines the $d$ band dispersion should be mainly responsible for the energetics. In our calculation, the overlap population indicates that the rib $M-\mathrm{P}$ bond in the $\mathrm{CaAl}_{2} \mathrm{Si}_{2}$ structure is always stronger and the handle bond weaker than the $M-\mathrm{P}$ bond in the $\mathrm{ThCr}_{2} \mathrm{Si}_{2}$ structure.

Figure 6 shows the energy difference between the two structures for two $M-\mathrm{P}$ separations ( 5 and $2.45 \AA$ ). A positive energy means the $\mathrm{CaAl}_{2} \mathrm{Si}_{2}$ structure is favored. These curves resemble those in Fig. 5. At electron counts close to $d^{0}$ or $d^{10}$,


Fig. 6. Energy difference between the $\mathrm{CaAl}_{2} \mathrm{Si}_{2}$ - and $\mathrm{ThCr}_{2} \mathrm{Si}_{2}$-type $\mathrm{Mn}_{2} \mathrm{P}_{2}^{2-}$ compounds as these are constructed by bringing up the phosphorus sublattice to the Mn sublattice. $\Delta E$ is defined as $E\left(\mathrm{ThCr}_{2} \mathrm{Si}_{2}\right)-E\left(\mathrm{CaAl}_{2} \mathrm{Si}_{2}\right)$. Thus a positive value means that the $\mathrm{CaAl}_{2} \mathrm{Si}_{2}$ structure is more stable than the $\mathrm{ThCr}_{2} \mathrm{Si}_{2}$ one.
the $\mathrm{CaAl}_{2} \mathrm{Si}_{2}$ structure is favored, partly due to the fact that fewer $M-M$ bonding states are pushed up by the $M-P$ interaction in the less dispersive metal sublattice in this structure. There is greater probability that an $A B_{2} X_{2}$ compound will fall into the $\mathrm{CaAl}_{2} \mathrm{Si}_{2}$ structure at a configuration $d^{0}$ or $d^{10}$. Interestingly, magnetic susceptibility measurements of $\mathrm{YbZn}_{2} \mathrm{P}_{2}$ and other compounds by Zwiener et al. (13) indicate that Yb has an intermediate valence between $\mathrm{Yb}^{2+}$ and $\mathrm{Yb}^{3+}$. In this case, the host lattice $B_{2} X_{2}$ need not possess a $d^{10}$ configuration.

Could we take the shape of the relative stability curve (Fig. 6) of the two structures as an indirect argument for the Mn compounds crystallizing in the $\mathrm{CaAl}_{2} \mathrm{Si}_{2}$ structure type as being high spin? Figure 6 is computed for a low-spin, paired electron assignment, and the region around $d^{5}$ is one
of maximum $\mathrm{ThCr}_{2} \mathrm{Si}_{2}$ structure stability. We do not have the energetics of electron repulsions and magnetic interaction in our calculations, so we cannot compare highspin alternatives with any confidence. But roughly speaking, the electron distribution in a high-spin $d^{5}$ compound is going to resemble an average of $d^{0}$ and $d^{10}$, which would return us to a region of $\mathrm{CaAl}_{2} \mathrm{Si}_{2}$ structural stability.

We have so far explored the nonmagnetic interaction that determines the relative stabilities of the $\mathrm{CaAl}_{2} \mathrm{Si}_{2}$ and $\mathrm{ThCr}_{2} \mathrm{Si}_{2}$ structures. It would be interesting to investigate, experimentally, magnetic properties of these marvelous compounds, particularly the $d^{5} \quad \mathrm{CaAl}_{2} \mathrm{Si}_{2}$ type. Our understanding of the peculiar requirement of $d^{0}, d^{5}$, and $d^{10}$ configurations in the $\mathrm{CaAl}_{2} \mathrm{Si}_{2}$ structure would be aided by such studies.

TABLE II
Extended Hückel Parameters

| Orbital |  | $\underset{(\mathrm{eV})}{\boldsymbol{H}_{\mathrm{ii}}}$ | $\zeta_{1}{ }^{\text {a }}$ | $\zeta_{2}$ | $c_{1}{ }^{\text {a }}$ | $c_{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Mn | 3d | -11.67 | 5.15 | 1.7 | 0.5140 | 0.6930 |
|  | $4 s$ | -9.75 | 1.8 |  |  |  |
|  | $4 p$ | -5.89 | 1.8 |  |  |  |
| P | $3 s$ | -18.6 | 1.8 |  |  |  |
|  | $3 p$ | -14.0 | 1.8 |  |  |  |

${ }^{a}$ Exponents and coefficients in a double $\zeta$ expansion of the $3 d$ orbitals.

## Appendix

The extended Hückel method (11) was used in all calculations. A rigid band model with fixed Mn and P parameters (listed in Table II) was chosen to represent the transition series. A Mn-Mn distance of $2.83 \AA$ and M-P $2.45 \AA$ was employed so that an ideal tetrahedral packing in both structures is realized. A $30 k$-point set is used in the irreducible wedge in the Brillouin zone to calculate DOS and COOP and total energies according to Ref. (12).

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